COEFFICIENT OF VARIATION FOR STRENGTH AND DEFORMATION PROPERTIES OF CLAY

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KEYWORDS

Coefficient of variation, clay, strength and deformation parameters

SUMMARY

Eurocode opens up for a probabilistic based design in geotechnical engineering. This means that there is a strong need for knowledge about the scatter of geotechnical parameters, such as strength and deformation parameters. In the paper the empirical correlation between the undrained shear strength and the preconsolidation pressure is illustrated through a number of cases from a comprehensive data base.
The coefficient of variation is then calculated for active, direct shear and passive undrained shear strength as well as for the preconsolidation pressure by using a Bayesian statistical method.

The coefficient of variation for these parameters is found to be substantially smaller than usually assumed. This is very important knowledge when the probability of failure shall be estimated for a slope or a foundation.

1. BACKGROUND

In the early days of Eurocode the ambition was a design philosophy based on a probabilistic approach. The probability of failure should be less than a given number or the probability that a settlement became larger than a reasonable value should be given.
However, as the knowledge of the scatter of the crucial geotechnical parameters was scares and also the simulation capabilities were rather limited, the method of partial factors of safety was introduced as a proxy method.

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During the last few decades, important progress has been made in the area of simulation, including stochastic parameters. Also, the use of advanced laboratory testing methods, such as triaxial testing and direct shear tests, has increased tremendously in Sweden.
In a separate research project, data were collected from over thirty test sites where high-quality triaxial tests, direct simple shear tests, and CRS (Constant Rate of Strain) oedometer tests were performed on clay samples. These tests constitute the database for the research presented in this paper.

The empirical relation for the ratio of the undrained shear strength and the preconsolidation pressure, as determined by the Swedish standard procedure for CRS tests, is presented in Fig. 1.

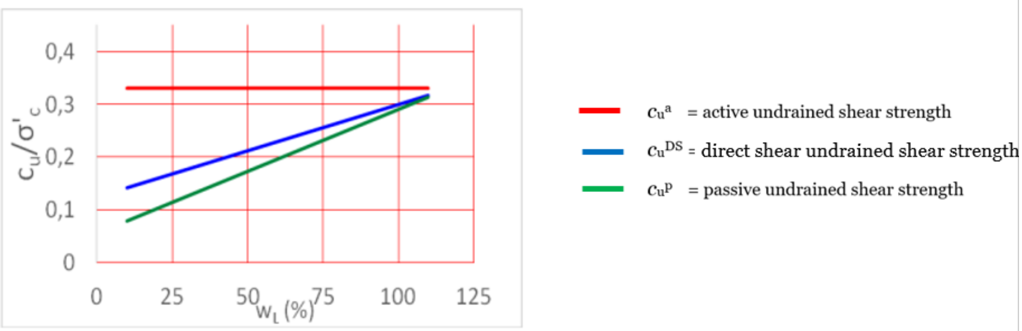


Figure 1. Empirical correlation between the ratio of the undrained shear strength and the preconsolidation pressure as a function of the liquid limit w_L. [4]

These relations refer to the shear strength for lightly overconsolidated clays. If the OCR is larger than 1.3, the strength should be adjusted for OCR according to eq. 1, to account for the overconsolidation of the sample before c_u/σ'_c is calculated and plotted in Fig.1.

$$\left(\frac{c_u}{\sigma'_c}\right)_{OC} \approx \left(\frac{c_u}{\sigma'_c}\right)_{NC} * OCR^{-0,25} \qquad \text{eq 1.}$$

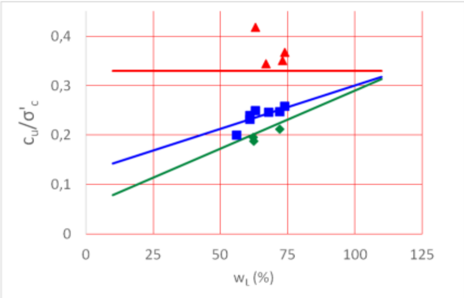
here c_u = undrained shear strength
σ'_c = vertical preconsolidation pressure
OCR = overconsolidation ratio

A database was established and consists of some thirty different cases, including the four different test methods mentioned above. This is discussed in the next section.

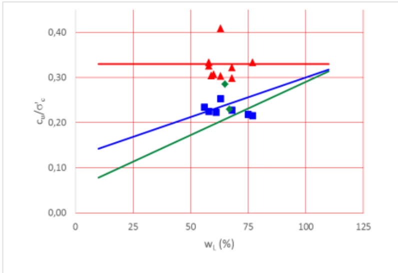
2. RESEARCHED MATERIAL

It is well known that there is a close correlation between the undrained shear strength and the preconsolidation pressure. This ratio c_u/σ'_c is usually presented as a function of the liquid limit. w_L. However, it is evident that the w_L should be replaced by K_o^{nc}, where K_o^{nc},= $\sigma'_{ch}/\sigma'_{cv}$. It appears logical as K_o decreases with decreasing w_L. Also, it is worth noting that the anisotropy, c_u^a/c_u^p increases with decreasing w_L and decreasing K_o. The lines in Fig. 1 clearly illustrate this.

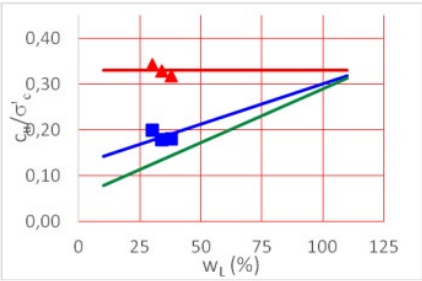
In a rather comprehensive research project, [1], dealing with geotechnical properties of Swedish clays, about 30 test sites were investigated where numerous CRS tests, direct simple shear tests and triaxial tests had been performed, the empirical relation between the undrained shear strength and the preconsolidation pressure was investigated. Here the results from CRS, direct shear tests and triaxial tests were available and the results were plotted together with the empirical correlation. A few examples are given in Fig.2 a – d.



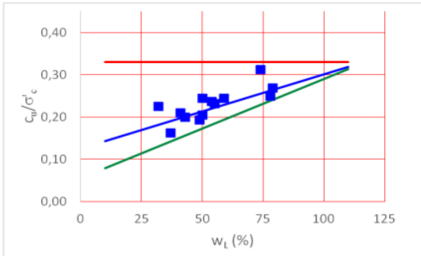
a. Karlatornet



b. Riksvägv45



c. Norsälven

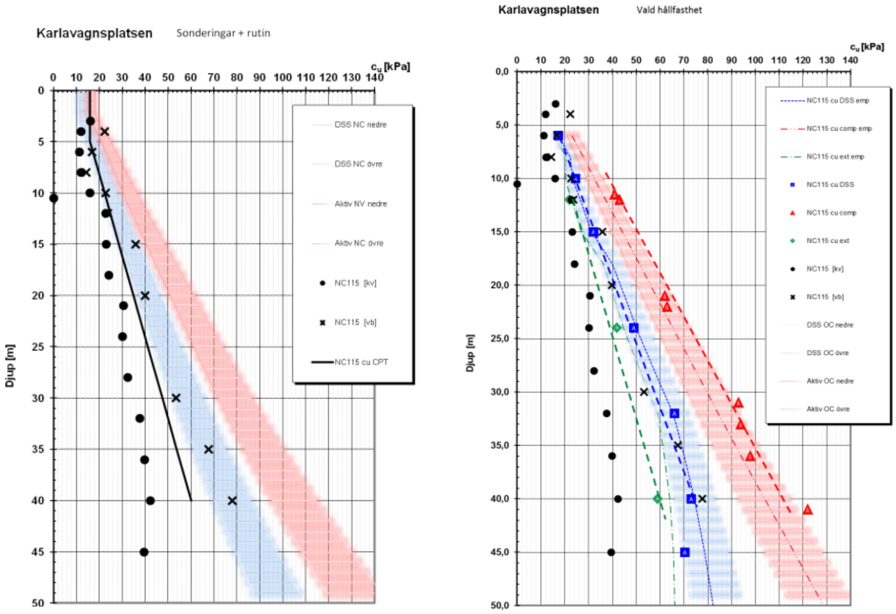


d. Uppsala

Figure 2. Results from a. Karlatornet, Hisingen, Göteborg. b. Riksväg 45
c. Norsälven, Värmland d. Uppsala
red line = active triaxial strength
blue line = direct shear strength
red line = passive triaxial strength

3. PROCEDURE FOR ESTIMATING THE UNDRAINED SHEAR STRENGTH, c_u, BASED ON THE VERTICAL PRECONSOLIDATION PRESSURE, σ'_c

In a geotechnical task where the undrained shear strength is of great importance, the recommended procedure includes plotting the preconsolidation pressure determined from CRS tests as a function of depth. A best fit line is chosen for the variation of the preconsolidation pressure with depth and, using the empirical graph described above, then results in an expected variation of c_u^a and c_u^{DS}, with depth, see Fig. 3.a. The blue and red areas in the figure indicate a +- 10 % range for the estimated shear strengths.



a. *b.*

Figure 3. a. Expected undrained shear strength, based on the empirical method
b. Empirical shear strength and test results
red line = active triaxial strength
blue line = direct shear strength

In Fig.3.b the actual results from triaxial tests and direct shear tests are also plotted. Finally the most probable trendlines for the different shear strengths are chosen, indicated by the dashed lines in Fig. 3.b.

It is important to bear in mind that empirical correlations are valid for an aged clay with horizontal ground, and which has not been exposed to leaching or external loading on the surface during the last couple of hundred years.

4. THE USE OF BAYESIAN STATISTICAL METHOD

When doing probability-based design, it is imperative to have not only a good knowledge of the trend with depth for the parameters, but also quantitative information about the scatter of the parameters.

The only systematic method for getting a reasonable estimate of the scatter of geotechnical parameters, as the number of test results is limited is through the use of Bayesian statistics.

In Bayesian statistics, prior knowledge of the parameter is used together with the new test data to derive at a posterior distribution for the parameter. This type of simulation uses MonteCarlo simulation, combined with Markow Chains (MCMC).

In this case, where the undrained shear strength is of interest, the vertical preconsolidation pressure variation with depth is first estimated based on stratigraphy and results from CRS tests. In this program the preconsolidation pressure is assumed to be constant down to a given depth k. From then on, the preconsolidation pressure is assumed to increase linearly with depth. By use of the empirical diagram, see Fig. 1, an estimate of the variation of c_u with depth is obtained. This variation of the undrained shear strength is taken as the prior distribution for the shear strength. Together with the shear strength values evaluated from the laboratory tests, the MCMC simulation is performed. This simulation then results in a posterior distribution of the shear strength variation with depth. The so-called posterior distribution of the undrained shear strength is also constant down to the depth k, and from then on increases linearly with depth. The coefficient of variation is then assumed to be constant with depth.

A computer program was developed using MCMC simulation and giving the best estimate of the trend line with depth for the undrained shear strength and the coefficient of variation for the shear strength.

The results from such a simulation are given in Fig. 4.

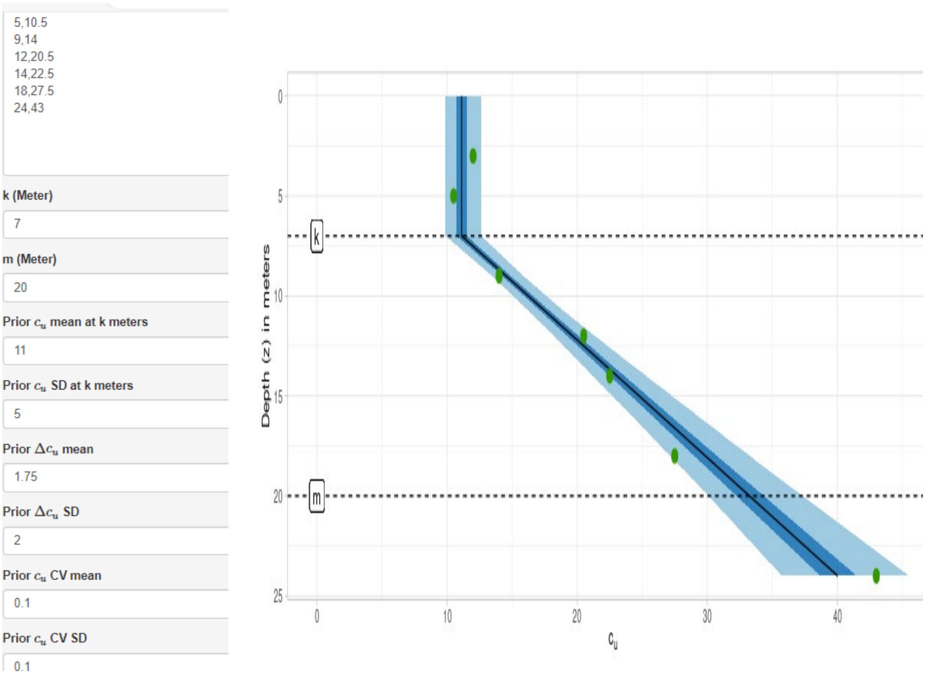


Figure 4. Results from an MCMC simulation of the undrained shear strength variation with depth.
The column to the left indicates the input for the simulation and the graph to the right shows the variation of the shear strength with depth.
dark blue field indicates a 50 % confidence limit and the light blue field indicates a 95 % confidence limit

As can be seen in the figure, the shear strength was assumed to be bi-linear with depth, and the variation of the preconsolidation pressure resulted in a prior distribution of the shear strength being 11 kPa down to 7 m and then increasing with 1,75 kPa/m.

The simulation resulted in a trendline being equal to 11,3 kPa down to 7 m and then increasing with 1,7 kPa/m. The coefficient of variation for the posterior function of the shear strength was estimated to be 6 %.

5. APPLICATION OF THE METHODOLOGY TO THE DATABASE OF UNDRAINED SHEAR STRENGTH TESTS

The procedure with the MCMC simulations was applied to the results from all the test sites mentioned above and was presented in [1]. This is a very comprehensive database and the simulations resulted in rather consistent results.

Apart from the trendlines, which showed some notable consistency, the coefficient of variation for each site and test type was also obtained from the simulation program.

The results were very consistent, and the coefficient of variation was in the order of 5 to 7% for the undrained shear strength and slightly higher for the preconsolidation pressure.

The authors are convinced, that if the geological sorting is made with care, the coefficient of variation for most clays in Sweden is around 6 % for c_u^a and c_u^{DS} as well as c_u^p and 7 % for the σ'_c, probably less.

In order to obtain such low values for the coefficient of variation for these parameters, it is very important that sampling is carefully performed, lab testing made with great care and the results are scrutinized and faulty values, or non representative values are omitted, just in line with what the Eurocode requires. It is especially important to consider whether the samples are undisturbed or not.

6. CONCLUSION AND POSSIBLE FUTURE DEVELOPMENT

The empirical correlation between the preconsolidation pressure and the undrained shear strength of clays, active, direct shear as well as passive, is illustrated and found to be very strong. Evaluation of the coefficient of variation is done utilizing Bayesian statistical methods and was found to be around 6 % for a large number of different sites.

ACKNOWLEDGEMENT

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