ON THE INFLUNCE OF SYSTEM STIFFNESS FOR NUMERICAL DESIGN APPROACHES

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KEYWORDS

Design approach, Eurocode 7, numerical simulation, FEM, excavation pit

ABSTRACT

The use of the Finite Element Method (FEM) to predict deformations at the serviceability limit state (SLS) has become established in geotechnical engineering over the past few decades. The revision of Eurocode 7 provides numerical methods to be used in the design procedures for the ultimate limit state (ULS). To this end, the Input Factoring Approach (IFA) and the Output Factoring Approach (OFA) are introduced. At present, it is not clear which boundary conditions are relevant for the different design approaches.

Research has focused on the influencing factors of excavation pit design for decades. As a result, the most important parameters for excavation pit design are the shear parameters (φ' and c'), the tensile stiffness EI of the wall and the soil stiffness E_s. The influence of different parameters on the design approach (IFA and OFA) for a single braced as well as a double braced excavation pit is analysed by means of a parameter variation. The design-relevant method is determined by applying known stiffness factors for the analysed systems. In conclusion, the results are discussed and an outlook on future research directions is outlined.

1. INTRODUCTION

In Germany the FEM is currently primarily used for predicting deformations in SLS. Therefore, numerical methods are applied at a relatively late stage in the construction planning process. The revision of Eurocode 7 introduces a normative regulation of the use of FEM for ULS design in geotechnical engineering for the first time [1]. Stability analysis will be conducted using two methods: Input Factoring Approach (IFA), which reduces shear parameters, and Output

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Factoring Approach (OFA), which factorizes characteristic internal forces resulting from numerical calculations. The analyses will cover both ground failure and structural failure of the geotechnical structure.

However, both approaches require time-consuming calculations that can be complicated by occurrence of various failure mechanisms or numerical instabilities. At an early planning stage, this may not be feasible due to limited ground information, and therefore established analytical methods are used. Therefore, it would be advantageous to use only one method in early stages of the planning process. To understand the design-relevant approach (IFA or OFA), it is important to consider the various influencing factors. Various authors have examined the factors influencing excavation pit design [2-6]. The present paper analyses the factors influencing the design approach using two synthetic excavation pits and applies stiffness parameters for the system from literature to analyse the design.

2. THEORETICAL BACKGROUND

In the design of geotechnical structures, it is necessary to factorize both actions and resistances with partial safety factors. However, due to the non-linearity of the soil material, this procedure cannot be carried out by means of numerical methods. To address this issue, both the IFA and OFA method, which consider both actions and resistances with partial safety factors (refer to Table 1), will be used in future. Numerical simulation has the advantage of taking into account the stress history. Nevertheless, the appropriate point in the calculation to factorise the input parameters is uncertain. One option is to apply the partial safety factors at the beginning of the simulation (IFA, variant A), while the other is to apply them in form of a bifurcation calculation during the design phases (IFA, variant B).

Table 1. Partial factors for IFA and OFA.

	OFA	IFA
Permanent action (G_k)	$\gamma_\mathrm{G} = 1.00$	$\gamma_\mathrm{G} = 1.00$
Variable action (Q_k)	$\gamma_\mathrm{Q}/\gamma_\mathrm{G} = 1.50/1.35$	$\gamma_\mathrm{Q} = 1.30$
Effects of actions (E)	$\gamma_\mathrm{E} = 1.35$	$\gamma_\mathrm{E} = 1.00$
Internal friction ($\tan\varphi'$)	$\gamma_{\tan\varphi} = 1.00$	$\gamma_{\tan\varphi} = 1.25$
Cohesion (c')	$\gamma_\mathrm{c} = 1.00$	$\gamma_\mathrm{c} = 1.25$

Previous research [2-6] has used numerous case studies to investigate the influence of behaviour on the deformation as well as the load-bearing capacity of excavation pits. However, due to the complexity of the soil material and the numerous boundary conditions, it is almost impossible to identify generalised dependencies. In soft soils, Clough et al. [3] define the base heave as the significant failure mechanism due to plastic flow in the area of the earth support. To contain the associated horizontal wall deformation $u_{\mathrm{x,max}}$, Clough et al. [3] define a system stiffness factor according to Equation 1 that ensures a factor of safety against basal heave FOS_{base}. The system stiffness requires the wall stiffness EI, average vertical support spacing $\bar{h}$ and unit weight of water γ_{w} as input parameters.

$$\frac{EI}{\gamma_{\mathrm{w}}\,\bar{h}^4} \qquad \text{Eq. 1}$$

Bryson and Zapata-Medina [6] present a generalised approach to analysing excavation structures. They compile all parameters that influence the safety of the excavation against basal heave, based on three-dimensional numerical analyses of a triple braced excavation pit. The relative stiffness ratio R, defined by Equation 2, can be classified into three groups. Firstly, the ratio of the wall stiffness parameters E and the in-situ soil stiffness E_{s}. Secondly, the geometric parameters above the excavation base in form of the average vertical $\bar{h}$ and horizontal support spacing s_{H}, the excavation depth h and the moment of inertia of the wall I. And thirdly, the parameters that affect the earth support area: average weight of the soil γ_{s}, undrained shear strength s_{u} and wall length H.

$$R = \frac{E}{E_{\mathrm{s}}}\,\frac{\bar{h}\cdot s_{\mathrm{H}}\cdot h}{I}\,\frac{\gamma_{\mathrm{s}}\cdot H}{s_{\mathrm{u}}} \qquad \text{Eq. 2}$$

3. NUMERICAL MODELS

Two synthetic systems are analysed: a single braced excavation according to Schweiger [7] and a double braced excavation according to Dahmen [8]. The numerical models' sections, including the discretisation, are shown in Figure 1. The mesh uses 15-node triangular elements with a target element size of $l_{\mathrm{e}} = 4.002$ m for system 1 (4,827 elements) and $l_{\mathrm{e}} = 3.002$ m for system 2 (2,497 elements), with local mesh refinement in the excavation pit area. The distance to the boundary of the model is selected to be large enough $(2-3h)$ to avoid any significant boundary effects.

The Hardening Soil constitutive model by Schanz et al. [9] is used to model the sandy soil, with parameter values presented in Table 2. To conduct the parameter study, the stiffnesses and strengths of the soil are varied using the parameters listed in Table 2, with the initial parameters highlighted in bold. The model does not include any hydraulic boundary conditions. The numerical model implements either an AZ 18-700 (System 1) or a Larssen 43 (System 2) sheet pile wall with elastoplastic material behaviour. Additional sheet pile walls, including AZ 36-700, AZ 48-700, and an 80 cm wide diaphragm wall for System 1, and Larssen 25, Larssen 430, and an 80 cm wide diaphragm wall for System 2, are also analysed in the parameter study. The struts exhibit elastoplastic material behaviour with a tensile stiffness of $EA = 493{,}500\ \mathrm{kN/m}$ for system 1 and $EA = 150{,}000\ \mathrm{kN/m}$ for system 2.

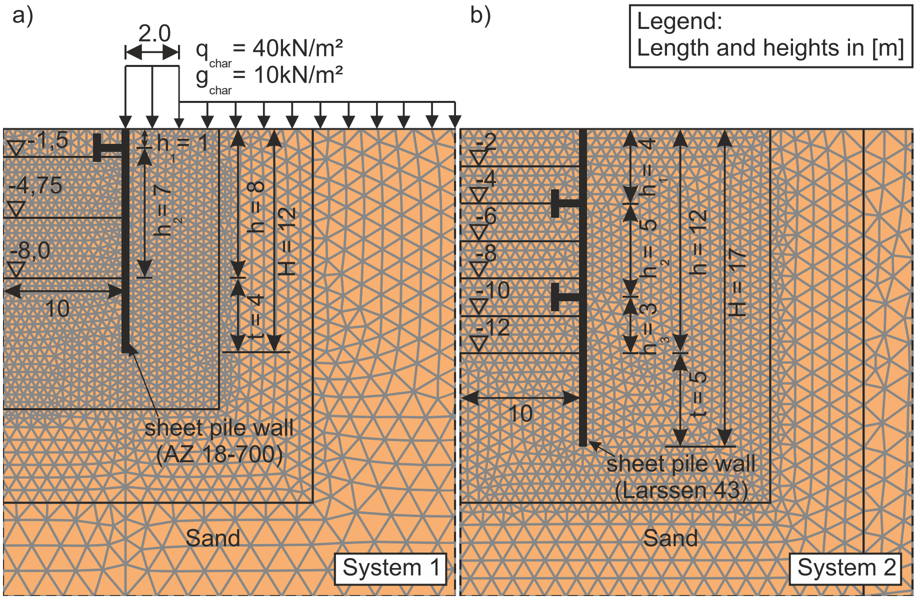

Figure 1 Numerical model for the single braced excavation (a) from Schweiger [7] and the double braced excavation (b) from Dahmen [8].

Table 2. Material parameters for the Sand (Hardening Soil model). Initial parameters are shown in bold.

γ_{unsat}	γ_{sat}	E_{50}^{ref}	E_{oed}^{ref}	E_{ur}^{ref}	m	ν_{ur}
$[\mathrm{kN/m^3}]$	$[\mathrm{kN/m^3}]$	$[\mathrm{MN/m^2}]$	$[\mathrm{MN/m^2}]$	$[\mathrm{MN/m^2}]$	$[-]$	$[-]$
18	19	**20**, 40, 60, 80, 100	**20**, 40, 60, 80, 100	**60**, 120, 180, 240, 300	0.5	0.2
φ'	c'	ψ	R_{inter}	p_{ref}	K_0^{nc}	R_{f}
$[°]$	$[\mathrm{kN/m^2}]$	$[°]$	$[-]$	$[\mathrm{kN/m^2}]$	$[-]$	$[-]$
25, **30**, 35, 40	**0.1**, 5, 10, 15	0	0.63	100	0.500	0.9

4. RESULTS OF THE PARAMETER STUDY

Figures 2 and 3 display the maximum and minimum bending moments (M) for various retaining walls, stiffness as well as strength parameters, in the final state of the excavation resulting from the design calculation according to IFA, variants A and B as well as OFA. In addition, the results for the final state with characteristic parameters are presented.

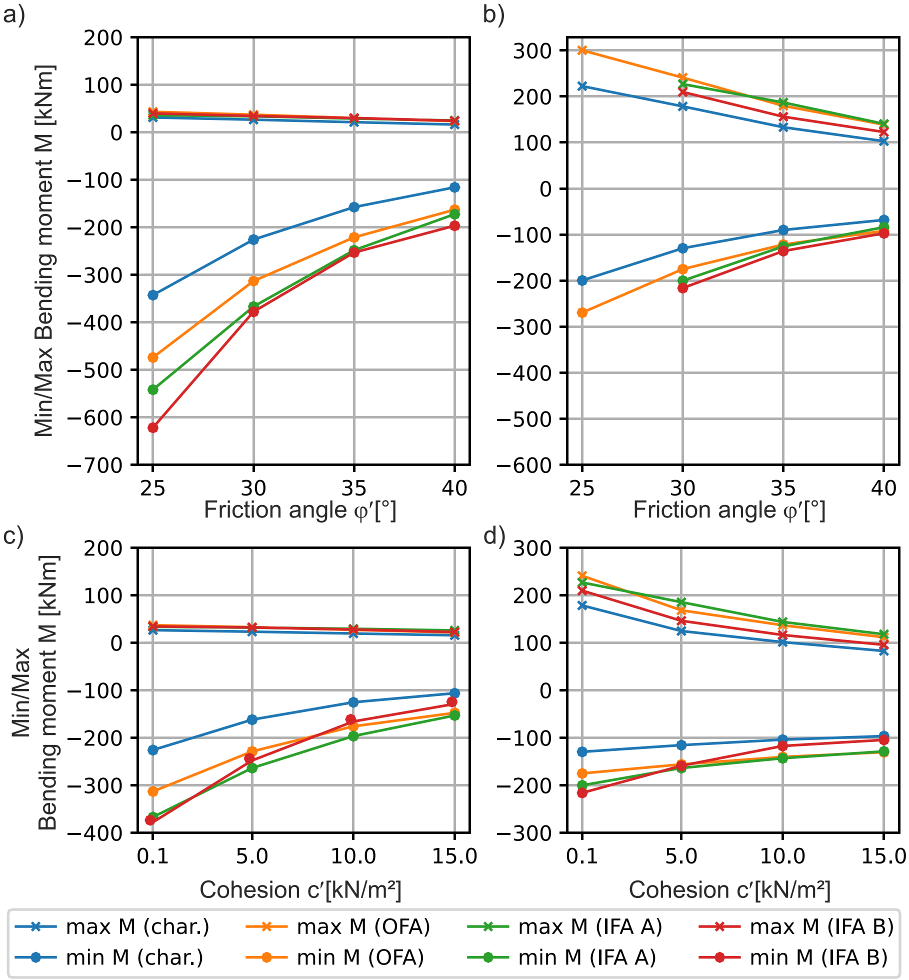

Figure 2 Maximum and minimum bending moments M for the varied values of the friction angle φ′ (a and b) and cohesion c' (c and d). System 1 left column and system 2 right column.

IFA, variant B is the dominant design approach for the systems 1 and 2 when varying the soil strength parameters (φ' and c') in Figure 2, as the minimum moment $M_{\min}$ is relevant for the design of the sheet pile wall. It is evident that the delta Δ in minimum bending moment M_{min} between IFA (variant A or B) and OFA decreases if soil strength increases.

Only for system 2 with $\varphi' = 30°$, the OFA calculation results in a slightly higher maximum bending moment M_{max} at the strut compared to the IFA calculation (see Figure 2b). This indicates that OFA tends to become design-relevant when a stiff component dominates the system, regardless of whether it is the soil or the strut.

The investigation of the different soil stiffnesses in Figure 3a and b indicates no significant influence on the design-relevant approach, with the oedometric stiffness E_{oed} serving as the representative value on the x-axis. Figures 3c and 3d demonstrate that different wall types (represented by the moment of inertia I_{y}) result in design-relevant bending moments in OFA design when wall stiffness is high.

The retaining wall's variation confirms the previously mentioned tendency of OFA as design-relevant method if stiff components are dominating the entire system. Additionally, the soil strength is significant in evaluating the design-relevant approach (IFA or OFA). Reducing the soil strength parameters in IFA simulations can be design-relevant if the soil is close to the limit state. In such cases, the soil can carry comparatively less load and the bending moments of the retaining wall increase with IFA as design-relevant approach. If the soil has sufficient load-bearing capacity, the bending moments in IFA design will remain small and OFA becomes design-relevant due to factoring the characteristic effects of action.

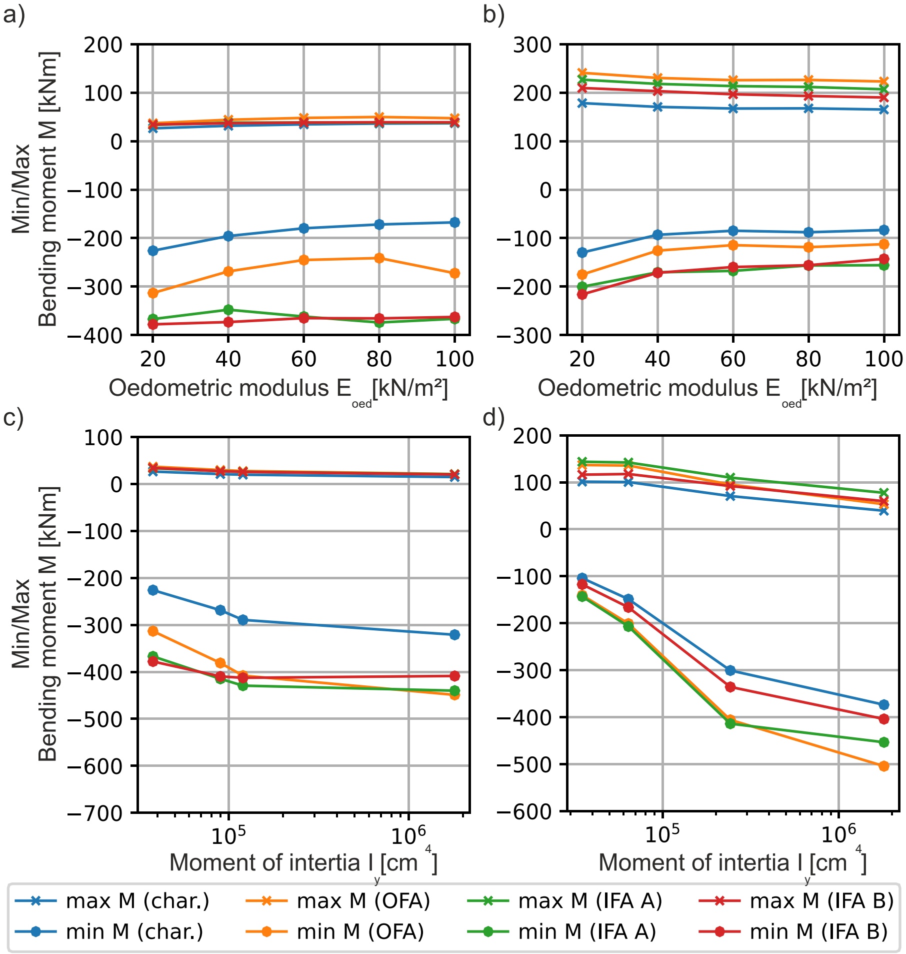

Figure 3 Maximum and minimum bending moments M for the varied values of the oedometric modulus E_{oed} (a and b) and moment of inertia I_y of the wall (c and d). System 1 left column and system 2 right column.

5. INFLUENCE OF THE SYSTEM STIFFNESS

The ratio χ, which compares the design bending moment M_d according to IFA (either variant A or B) with OFA, is defined to analyse the influencing factors according to Equation 3. A ratio $\chi > 1$ means that the relevant design results considering the bending moment of the retaining wall are based on the IFA calculation whereas for $\chi < 1$ they are based on the OFA simulation.

$$\chi = \frac{\max\left(\left|M_{\mathrm{d,IFA\ A}}\right|\ ;\ \left|M_{\mathrm{d,IFA\ B}}\right|\right)}{\left|M_{\mathrm{d,OFA}}\right|} \qquad \text{Eq. 3}$$

In both excavations analysed, the system stiffness is calculated using the unit weight of the soil γ_s instead of the unit weight of water γ_w, as hydraulic conditions are not applied. In addition, the numerical models are analysed considering drained conditions, resulting in the conversion of undrained shear strength s_u using the approximation in Equation 4.

$$s_\mathrm{u} \approx \tau = \frac{\sigma_1' + \sigma_3'}{2} \cdot \sin\varphi' + c' \cdot \cos\varphi' \qquad \text{Eq. 4}$$

Figures 4 and 5 show the ratio χ for the parameter variations carried out in section 4 for systems 1 (orange) and 2 (green) as a function of the respective system stiffnesses according to Clough et al [3] and Bryson and Zapata-Medina [6].

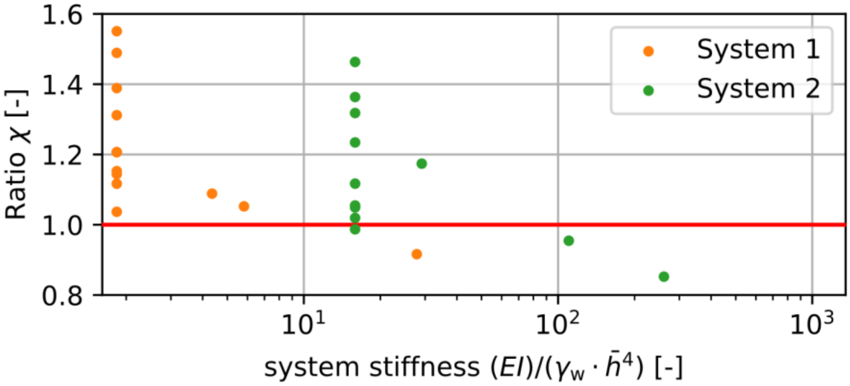

Figure 4 System stiffness using the varied parameters for system 1 and 2 in relation to the ratio χ.

The investigation of the system stiffness according to Clough in Figure 4 shows that a higher bending stiffness EI of the retaining wall leads to a stiffer system behaviour, with ratio $\chi < 1$ and OFA simulations becoming design-relevant. However, the variation of soil strength parameters (φ' and c') has not been considered, and the system stiffness is only partially identical, even though the values for the ratio χ vary. Although the influence of strength parameters on the design-relevant method (IFA or OFA) is relatively minor (see Figure 2), it is important not to ignore this aspect when considering the system stiffness.

Figure 5 illustrates the use of the more general approach by Bryson and Zapata-Medina [6] to consider the strength parameters. The parameter variations that determine the decisive design moment as a result of IFA are in a similar range (R > 17.5). Only system 2 shows an outlier, where OFA is identified as design-relevant method. In this case, the parameter variation $\varphi' = 30°$ and $c' = 15\ \mathrm{kN/m^2}$ is applied, indicating that the soil has a comparatively high shear strength. The factor of safety $FoS = 1.878$ for the final state due to strength reduction is within an above-average safe range. Therefore, a construction other than the double braced excavation pit would be more economical and the system selected here is not optimal.

In principle, however, it can be stated that the relative stiffness ratio *R* according to Bryson and Zapata-Medina [6] allows a tendency to predict the design-relevant method (IFA or OFA) for the systems analysed here.

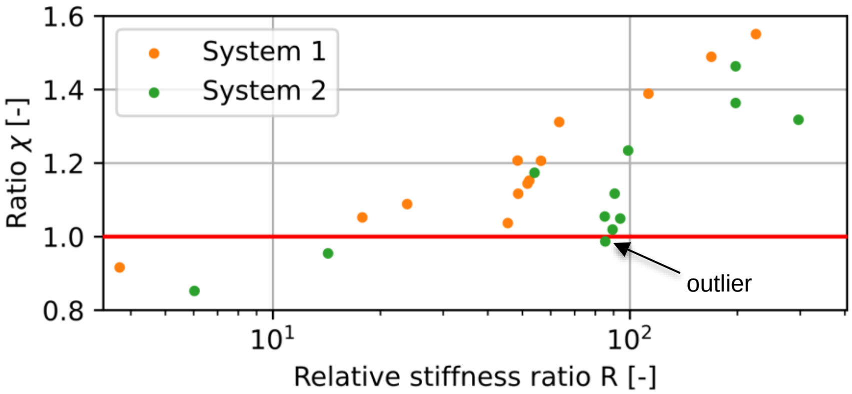

Figure 5 Relative stiffness ratio R using the varied parameters for system 1 and 2 in relation to the ratio χ.

6. CONCLUSION AND OUTLOOK

This paper analyses the numerical design methods IFA and OFA for single and double braced excavations. The study investigates the influence of various parameters on the bending moment of the wall and highlights the dependency on the design approach. The results indicate that OFA is more relevant for design when a stiff component dominates the entire system, as these components can absorb greater stress/load. If the soil is close to the ultimate limit state and the wall must absorb the necessary loads, a reduction in strength parameters by means of IFA design becomes relevant.

To predict the relevant design approach, the system stiffness according to Clough et al. [3] and the relative stiffness ratio *R* according to Bryson and Zapata-Medina [6] are applied to the two systems as a function of the varied parameters. It is shown that the relative stiffness ratio R can be used to determine whether IFA or OFA method is appropriate for design purposes. This allows for better use of numerical methods in early stages of excavation planning to analyze stability in ULS.

Future investigations will include synthetic systems that consider hydraulic conditions and layered soil. Furthermore, the investigation will encompass real excavation pits and systems from literature.

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