IMPLICATION OF 3D SLOPE STABILITY ANALYSES FOR SENSITIVE CLAY SLOPES

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KEYWORDS

Slope stability, sensitive clays, 3D-analysis, safety factors

ABSTRACT

Stability analyses in sensitive clays is commonly carried out assuming a 2D plane strain condition and limit equilibrium (LE) approach using e.g. the method of slices or the finite element method (FEM). However, there may be instances where these key assumptions may not be representative. In such cases, it is important to quantify the 3D effect on slope stability and to make considerations for the post-peak reduction in undrained shear strength of highly sensitive clays. This work investigates current approaches adopted to address 3D effects in the current Norwegian practice and their implication for highly sensitive clay slopes. These effects are illustrated using 2D and 3D numerical analyses. An emphasis is made on highlighting aspects that play key roles in analyses results. Typical variations of key parameters are studied to make basis for sensitivity analyses. Different slope widths were analysed, and results demonstrated that the width of the 3D model controls the width of the failure surface and thus also the factor of safety. An attempt is also made to back-calculate slope failure using a strain softening material model in 3D FEM. Sensitivity study showed that the spatial variability of the soil strength has significant effect on the calculated safety factor. The work discusses challenges and limitations that could be encountered in analysing natural slope with current approach for 3D. Implications of these results considering current design code requirements is discussed.

1. INTRODUCTION

Slope stability is a commonly encountered problems in geotechnical engineering practice. Stability analyses is commonly carried out based on two-dimensional (2D) limit equilibrium methods (LEM) with the method of slices. A key assumption in these approaches is that the stability analyses is solved assuming a plane strain condition (2D) and an equal level of strength mobilization along a pre-selected slip surface using the method of slices. Such approach is commonly adopted in practice to

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perform slope stability analysis. Still, there are several instances encountered in projects where 2D assumptions may not be representative for various reasons such as the geometry of the slope, soil layering, loading conditions as well as extent of planned construction activity. In such cases, it would be appropriate to do a three-dimensional (3D) modelling of the problem. It is particularly important to take into account 3D effects in back analysis of slopes to avoid overestimation of shear strength (e.g. Stark and Eid 1998; Akthar 2011; Jostad et al. 2021). There has been extensive research going on developing ways to perform 3D slope stability analysis based on three broad categories of LEM, limit analyses methods and stress-strain methods (see summary in Akthar 2011; Zhang 2013, Kumar et al. 2023). In Norway and Sweden, there has been recent works looking at the effect of 3D slope stability problems (e.g. Jostad and Lacasse, 2015; Solli 2020; Edstam 2021; Jostad et al. 2021).

There is huge interest towards incorporating 3D effects into designs. This work investigates current approaches adopted in the Norwegian practice. Implication of these is illustrated using numerical analysis involving 2D LEM and FEM analyses as well as 3D FEM analysis. Further emphasis is made on aspects that play key roles in the analyses results. The work aims to investigate the current state of the practice and point key and relevant aspects related to use of 3D effects on sensitive clay slopes.

2. MOTIVATION AND PROBLEM FORMULATION

As one of the main regulatory authority bodies in Norway, the Norwegian Public Roads Administration (NPRA) is, among others, responsible for issuing acceptable partial factors to be used in design of slopes associated with road infrastructures. The current safety requirements are developed and calibrated using 2D approach; thus, with a view to be used in 2D analysis. However, with emergence of powerful computers and technological advancements, there is increasing demand and development towards use of three-dimensional tools in Geotechnical engineering (e.g. BIM-models and Digital Twins). As a result, there is also interest on development and use of calculation tools for 3D slope stability analysis which affects the resulting safety factors. Therefore, it is important to investigate implication of this with a view to complying to current level of safety requirements when performing 3D stability analysis.

Independent of 2D or 3D modelling, an important feature of Norwegian sensitive clays is their strong strain-softening behaviour, i.e. post-peak shear strength reduction with deformation. Currently, this is addressed by requiring higher safety factors, for strain-softening materials, along with 2D LEM stability analysis tools (Fornes and Jostad 2017, NVE 2020). However, this effect is not thoroughly investigated and established for a 3D case. Therefore, it is important to address it with respect to 3D modelling.

In the current practice, there is growing tendency to benefit from higher calculated 3D safety factors while adhering to the safety requirements calibrated for use with 2D analyses (Degago, 2019). When this is used for sensitive clays, another challenge is that the effect of strain-softening is not quantified for a 3D case. These effect needs thorough assessment and to address this, a collaborative research project was initiated by NPRA and NGI since 2020 (NGI, 2022).

3. CURRENT INDUSTRY PRACTICE TO ACCOUNT 3D-EFFECT

In the current Norwegian practice, 3D-effect ($F3D = F_{3D}/F_{2D}$) is mainly calculated using two methods. The first one is use of a "3D Side Friction" function in GeoSuite Stability and the second one is using the finite element package Plaxis 3D (PLAXIS V21). In this section, the two approaches are briefly investigated numerically from application point of view into real projects.

GeoSuite 3D Side Friction

The approach adopted in the GeoSuite Stability is based on recommendation by the Swedish commission on slope stability (Skredkommissionen, 1995) which in turn bases itself on the work by Gens et al (1988). This method is developed assuming a simplified slope with homogeneous, isotropic, and purely cohesive soils. It further assumes rotational slides with a cylindrical central slip surface enclosed with plane sides; and this analytical expression is extended to consider curved ends. Another key assumption is that the boundary outside of the failed soil mass is strong enough to provide support for the end failure surfaces. The simplified theoretical development by Gens et al. (1988) provided the basis for adaptation by Skredkommision (1995) as given in Eqn. [1] with a later suggestion for minor correction (Edstam, 2021) as shown in Eqn. [2]. Both are given for the sake of completeness; otherwise, Dehlbom et al. (2021) showed that the difference between results of the two equations is practically marginal. In this approach, F_{3D} is calculated based on the critical F_{2D} and a safety factor for cylindrical failure volume, F_P , with slide length L and plane ends on the side.

$$F_{3D} = F_{2D} + 0.75 (F_P / F_{2D} - 1)$$
 [1]

$$F_{3D} = F_{2D} + 0.75 (F_P - F_{2D})$$
 [2]

As shown in Eq. [1] and [2], the input needed to calculate 3D safety factor, over a 2D slope stability analysis, is the length of the cylindrical failure surface (L). This is the input in GeoSuite stability to incorporate the so called "3D Side Friction".

To get the feeling of side friction implemented in GeoSuite stability, a comparison is made with 3D FEM analyses by Jostad and Lacasse (2015). Jostad and Lacasse (2015) used the FEM package Bifurc and studied 3D effects for a slope with constant undrained shear strength and provided a chart for various slope geometries. For simplicity, an isotropic constant undrained shear strength of c_u =40 kPa and unit weight γ =20 kN/m³ were used. The slope width (W) was varied in relation to the slope height (H). In addition, b = slope inclination from toe and d = D/H where D is the depth from the toe level to the bottom fixed boundary. Equivalent analysis is done with GeoSuite and presented along with 3D FEM results as calculated by Jostad and Lacasse (2015), Figure 1. This example is a simplified case involving a single isotropic soil with a constant strength and should be suited for comparison with GeoSuite. However, the results indicate that, the Side Friction formulation as adopted in GeoSuite stability gives higher 3D effect.

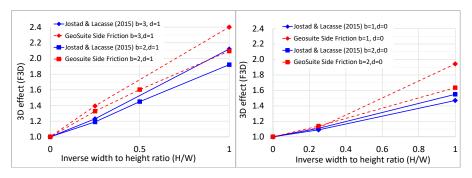


Figure 1. Implied F3D according to 3D FEM Bifurc and GeoSuite side friction (W= slope width, H= Slope height, b = slope inclination, d = (depth to strong layer)/H)

The practical implication of the Side Friction approach is further elaborated using real example, see Figure 2. Unlike back-calculation of failed slopes, design requires pre-evaluation of parameters. Therefore, in practice, defining the length or extent for the 3D failure surface could be challenging. This is illustrated using analysis on a sensitive clay slope from a road project in Norway with average slope heigh of 15 m. The two-dimensional analyses gave $F_{2D}=1.29$ and assuming different length for side shear (Side Friction) gave varying increase in safety factor, see Figure 2. Assuming various length for 3D side friction has significant implication on the calculated factor of safety (FOS). It is worthwhile to note that GeoSuite stability does not change the location of the critical slip failure in the 2D profile. In reality, it is expected that the depth of the 3D failure surface differs from the corresponding 2D failure line.

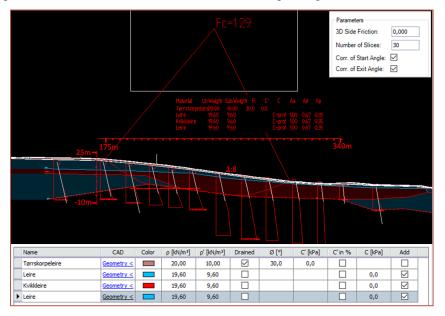


Figure 2 Geosuite stability analyses: Application example from a real project.

In the chosen example, the required FOS for the slope was 1.4 and this is not satisfied in 2D analysis as it gave $F_{2D} = 1.29$. However, one can easily show higher calculated

FOS by simply choosing side friction length up to 200 m (see Table 1). The key question is if it is correct to still aim for the same required FOS of 1.4 despite benefiting from side friction.

Table 1. Implication of 3D Side Friction length in safety factor (SF) in GeoSuite

Length for 3D effect, L (m)	50	100	200	400	∞
Calculated SF with Side Friction (-)	1.73	1.51	1.40	1.34	1.29
3D effect $(F3D = F_{3D}/F_{2D})$	1.34	1.17	1.09	1.04	1.00

Slope stability using FEM Plaxis 3D

FEM is a powerful numerical tool that is based on simulation of stresses and strains in a boundary value problem. Despite being computationally demanding, it has an advantage of simultaneously providing safety factor and identifying critical failure surface when applied to stability problems. For FEM applications, Plaxis 3D V24 is a commercially available tool commonly used in the industry; hence, selected in this work for numerical illustration.

The implications of 3D effect are investigated by using the geometries adopted in Figure 3. Two sets of shear strength profiles are selected for illustration of 3D effects. The first one is based on an idealized constant undrained shear strength of c_u = 40 kPa and the second one is a more realistic scenario where a shear strength profile increase with depth is depicted with c_u =20 kpa + 3 kPa/m. The geometry selected for analysis is a slope with a height of H=10 m; inclination 1: b =1:3; d=D/H=1 where D is depth to a strong soil layer or bedrock; and width W= 4H. For computational efficiency, symmetric modelling is performed. The sloping surface is modelled with ten soil columns each with a width of 3 m. This is a pragmatic way to enable varying reference level for undrained shear strength increase with depth in the sloping part of the slop terrain, see Figure 3. The 3D stability analysis is supplemented by 2D FEM analyses to provide a reference and a 3D plane strain analyses to quantify F_{3D} and effect of element discretization. The resulting factor of safeties as well as incremental displacement at failure for the 2D, 3D plane strain and 3D models are given in Figure 4. A relative comparison of analyses results shows that the 3D effect is higher for a constant c_u as compared to the most realistic scenario of increasing c_u with depth. This is logical as the volume of failure mass subjected to the side shear of the sliding surface becomes significantly higher for the case of constant c_u and the "side shear" has different average shear strength compared to the plane strain case. Further analyses of the results are presented along with element type and mesh discretization aspects in the next section.

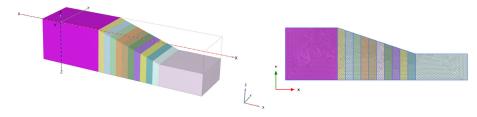


Figure 3 Geometry and soil layering adopted for 3D (left) and 2D (right)analyses.

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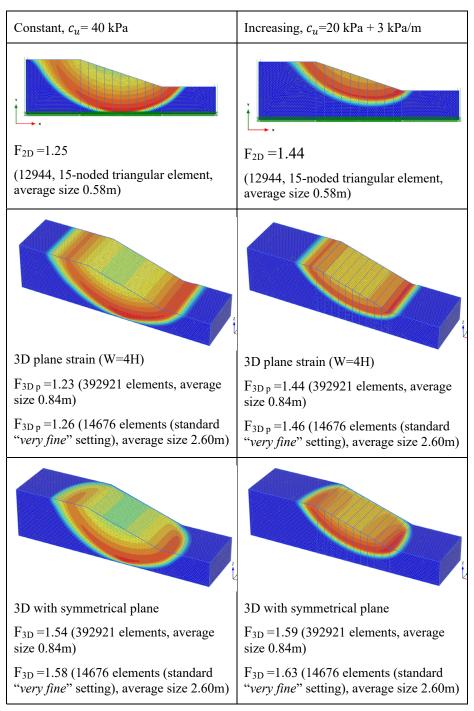


Figure 4 Incremental displacement at failure for 2D, 3D plane strain and 3D models.

Effect of mesh discretization and element type in Plaxis 3D

Mesh discretization and element types are important for analyses in FEM and particularly for 3D modelling. Plaxis 3D uses 10-noded tetrahedral elements with 4 Gaussian points and with second-order interpolation for displacements. Zhang et al. (2013) showed that hexahedral elements are more accurate and much more efficient than tetrahedral elements for stability/capacity analysis. Tetrahedral elements require huge number of elements and take much longer time to provide a factor of safety equivalent to those obtained by hexahedral element. Otherwise, for a given discretization, tetrahedral element tends to overestimate the calculated safety factor. This is also observed in the current analyses. Jostad and Lacasse (2015) analysis of the case with b=3, d= D/H =1, w=W/H=4 using a mesh containing 1,392 20-noded brick elements with (2x2x2) reduced Gaussian gave $F_{3D} = 1.55$. Equivalent geometry modelling in Plaxis using the standard "very fine" mesh setting gave 14,676 10-noded tetrahedral elements (with average size of 2.60 m). For this case, safety analyses showed a convergence issue giving safety factor of 1.58 after 100 calculation steps. Analysis with 392, 921 elements (average size 0.84m) gave $F_{3D} = 1.54$. The results are dependent on mesh discretization requiring huge number of elements to improve accuracy. This demonstrates the role of element type used in Plaxis for 3D stability analysis.

Effect of varying slope width on 3D factor

A series of highly refined additional numerical analyses were performed to study effect of varying slope width on the 3D stability. This is done for a soil material with constant and increasing c_n with depth. For a slope with height H = 50 m, the considered widths were H, 2H, 3H and 6H. Other fixed features of the geometry of the slope are b=2.5 and d=1.0. For all the analyses, an attempt was made to maintain an element size of 30 cm. To make the comparison of 3D effects, the corresponding 2D model should have same discretisation error as in 3D model. Therefore, all 2D cases were also simulated in Plaxis 3D ensuring plane strain boundary conditions. Two strength profiles are selected for comparison, i.e. constant $c_u = 35$ kPa and increasing c_n =20 kPa + 3 kPa/m. Further details of the model can be referred to NGI (2022). Summary of the analyses results are presented in Figure 5. The results show that 3D effect is higher for constant c_n and narrower slopes giving 80% increase for H/W=1. This reduces dramatically to 20% when increasing c_u with depth is considered. For this example, for $w^{-1} < 0.5$, the 3D effects become less than 10%. This trend implies that a realistic scenario of width of commonly encountered natural slopes and strength profile could rather have a limited 3D effect as compared to constant shear strength. The observed trend is also valid for drained slope stability analysis.

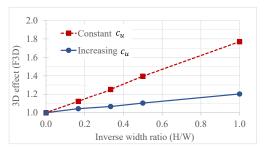


Figure 5 Variation of 3D effect factor with width of slope.

4. ON EFFECT OF STRAIN SOFTENING

A key aspect of highly sensitive clays is their post-peak reduction in undrained shear strength or strain softening behaviour. For 2D analyses, this aspect is indirectly accounted for by requiring higher safety factor (Fornes and Jostad 2017). An important feature of slope failures in highly sensitive clays is that a small initial slide can trigger and develop into fast and extensive landslide. The idealisation adopted in earlier section may not be suited to study strain-softening behaviour as local triggers initiating failure may not correspond with the rest of the slope geometry. Therefore, this effect is better studied with back-calculation of a failed slope consisting of highly sensitive clays.

Jostad et al. (2021) studied the Skjeggestad landslide, Figure 6a, with respect to 2D and 3D stability. The work demonstrated that one needs to consider 3D effect to real-istically explain the observed failure. At the same time, the strain-softening effects in 3D were not accounted for in the study by Jostad et al. (2021). The strain-softening aspect was studied in NGI (2022) with Plaxis 3D using a constitutive soil model referred to as NGI-ADP Soft. The NGI-ADP Soft is extension of the standard NGI-ADP soil model (Grimstad et al., 2012; PLAXIS V21). The NGI-ADP Soft can simulate strain-softening behaviour observed in highly sensitive clays.

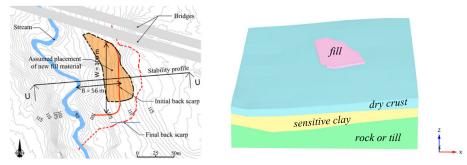


Figure 6 The Skjeggestad landslide a) slide extents (Jostad et al. 2021) b) Plaxis 3D model (NGI 2022)

A representative 3D model of the soil layering where the Skjeggestad landslide occurred was created in PLAXIS 3D based on kriging data, Figure 6b. A three dimensionally varying undrained shear strength profile for the sensitive clay layer was generated by interpolating between existing borehole data from the Skjeggestad landslide area. The 3D effect was analysed by comparing results of 2D and 3D calculations. In addition, the work looked at sensitivity of factor of safety to variation of undrained shear strength profile of the sensitive clay layer, strength of dry crust, rate of softening of the sensitive clay and volume of the fill placed on the slope, that is defined as the trigger for the Skjeggestad slide.

The 3D analysis with strain softening showed 12 % 3D effects for Skjeggestad slope as compared to the critical 2D profile. However, the stability of the slope was highly dependent on the spatial variation of undrained shear strength in the clay. Calculations with various strength profiles for the crust layer, demonstrated the need for extensive field and laboratory tests to accurately estimate the characteristics of the layer as it was found to significantly affect the safety of the slope.

5. DESIGN CODE IMPLICATIONS

In the current Norwegian practice, sensitive clay slopes are analysed assuming 2D condition. In addition, strain-softening behaviour is not directly assessed. Instead, the analyses are idealised using perfectly plastic behaviour with increased requirements to the calculated FOS. The required increase has been calibrated with extensive 2D numerical analyses comparing behaviour at calculated failure with softening and compared with calculated FOS without softening (NIFS 2015, Fornes and Jostad 2017). It is typically required to satisfy 15% higher safety factor to make provision for softening-behaviour (NVE 2020; NPRA 2022).

As previously mentioned, there is increasing interest for modelling slope stability with 3D as many believe that 3D analysis always gives higher safety for natural slopes (without construction). It is possible to numerically show higher safety factor with 3D effect. However, there are two challenges to this aspect. The first one is that the strain-softening effect in 3D is not thoroughly studied as in the 2D case. The second one is that adding a third dimension into the analyses introduces additional random variable that needs to be accounted for its uncertainty.

Stark and Ruffing (2017) argue that FOS calculated using 3D methods cannot be directly compared to 2D regulatory requirements. This is due to various contributing factors to uncertainty in slope stability analyses including inherent variability of input parameters, difficulty or quality in measuring input parameters, improperly selected critical failure surface, and limitations of stability analyses to represent complex subsurface conditions. As a result, Stark and Ruffing (2017) suggested a higher required safety factor for 3D analyses as a function of level of uncertainty in the soil strength and consequences of failure.

There are key research works attempting to look into effect of spatial variability in connection to 3D modelling (e.g. Hicks et al. 2014; Hu et al. 2022, Varkey et al. 2023, Wu et al. 2024). Wu et al. (2024) concludes that even though 2D generally seems to be more conservative, for a geometrically uniform case, there are cases where due to spatial variability (in 3D) a 3D slide can happen that is not identified in 2D analyses. This implies that a 3D analysis might increases uncertainty (standard deviation). For an example, case, Varkey et al. (2023) demonstrated that a specific correlation length (in the horizontal plane) lowers the mean FOS with approx. 4% and increases the standard deviation of FOS with 0.06. While the case on only spatial variation in the 2D is considered (correlation length into the plane zero or infinite) the mean value of FOS is not significantly affected. However, standard deviation is highest for the highest correlation lengths. Considering such reduction and increased standard deviation as a possible scenario, a deterministic FOS from a 3D analysis where uncertainties are not considered would require a higher calculated FOS to ensure same probability of failure as the 2D case. Until this is further clarified, only including the positive aspects of 3D should be done with the utmost care.

Current safety factors are calibrated with 2D approach and are meant to be used with 2D analyses. It is not quantified how much of strain-softening and 3D are included in the current design requirements. As a result, this might require a new design philosophy along with a through and systematic calibrations.

6. FINAL REMARKS

The industry is recognizing the need to consider 3D effects in design when 2D idealization is not sufficient. As a result, increasing use of 3D is already being observed in the current practice. Primary focus of this work is stability of natural slopes consisting of highly sensitive clays. For 3D stability analyses of such slopes, it is important to quantify the contribution of side shear while also making considerations for the post peak reduction in undrained shear strength. This dual effect needs thorough study.

Numerical tools commonly used in the Norwegian practice are numerically investigated. Hence,

- The Side Friction approach adopted in GeoSuite overestimates the safety factor even for idealised and simplified case it is meant to be used. Its underlying assumptions could be too simple for a natural slope that exhibits strength increase with depth, anisotropy and involve frictional soil layers. Like all other numerical idealization, it is crucial to see if the underlying assumptions of the approach are applicable for the problem to be used. With calibration to a rigorous approach, one can use the approach for preliminary assessment of 3D effects for local construction activities, e.g. stability of section-wise excavations or fillings and stability of soil mass within lime cement ribs.
- Plaxis 3D gives better alternative to model 3D stability. It requires highly refined mesh to make a realistic prediction and avoid overshoot of the calculated safety factor, i.e., numerical convergence issue giving higher safety factor. It is important to systematically study discretization error on the obtained safety factor and the corresponding mesh settings.
- For slopes in highly sensitive clays local triggers have huge significance in starting initial slide that can quickly spread to a much larger slide. Local trigger has little or no relation to bigger slope geometry and may not mobilise additional strength from 3D effects. Within the current state of knowledge, it is not recommended to rely on 3D effect for design in natural slopes of highly sensitive clays.

In general, one cannot aim to only increase calculated safety factor with 3D analyses without making provision for sources of uncertainty that follows 3D considerations. In such cases, the uncertainty in spatial variability of the slope in the third dimension needs to be considered. Current safety factors are developed and calibrated with respect to 2D analyses. Extensive and systematic work needs to be done to make corresponding calibration with consideration for 3D analyses.

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